Study of Vibration Specifications of a Three-axle Truck Using Lagrange Method

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Abstract
In this research, Ride quality of a three-axle truck is investigated to understand its vibration specifications and influence of it on special load. The truck is Benz 2624, and the load is rather sensitive to vibrations. The system is considered for an off-road duty. Hence, the vehicle is modeled as a nineteen-degree of freedom system and its equations of motion are derived by employing Lagrange equation. Since, the physical parameters of the vehicle were not available; the truck is modeled in Solidworks CAD software in order to obtain the material and dynamic properties of each component of the truck. Then, a code is developed in MATLAB to calculate natural frequencies and mode shapes of the truck and their corresponding vibrating components in critical speed. A simple model of the truck in ADAMS is employed for validation. The adoption of the results verifies the equations. The developed model can also be used in newer truck with some modifications. It is also necessary to have accurate data for input information in order to change the current model for other utilities.

Keywords: Vibration Analysis, Mode Shapes, Lagrange Equations, Multi-axles Truck.

1. Introduction
Modeling is an important part of engineering. There are two types of the modeling: Physical and numerical. Both types are widely used in engineering [1-8]. Vibration modeling and analyzing are of the application of modeling in engineering. Vibration analysis contributes to improve in many fields and products, e.g., aerospace, automobile, transportation, and so on. The most common goal is unwanted vibration’s identification and suppression to improve product quality. Multi-axel truck is a real world example, which requires its vibration breakdown.

Generally, vibrations are classified as free and forced vibrations that natural frequencies and mode shapes are the characteristics of the first one [9]. To obtain natural frequencies and mode shapes for damped vibration, two spaces are employed, time domain and frequency domain [10-13]. It is well-known that for a system, undamped and damped natural frequencies are very close particularly in case of vehicles dynamics because of small damping coefficient. Hence, undamped natural frequencies are commonly used to characterize the system [14]. Many approaches have been reported to extract natural frequencies with their own advantages and disadvantages, for instance, Prony method, generalized pencil-of-function method, matrix pencil method, higher order tensor-based method and Newton method [15-22].
Many models such as quarter, bicycle, half and full models of vehicle with different numbers of DoF have been investigated in vehicle dynamics [23-26]. One of the most famous models for vehicles is eight-DoF model, including forward, lateral, yaw and rolling motion plus, four degree of freedom for travel of each wheel [27-28]. Multibody system dynamic models of vehicles have also been proposed in the literature. For instance, RahmaniHanzaki et al. proposed a methodology for dynamic analysis of a multibody system with spherical joints. They considered a suspension system of a vehicle as an example for that [29]. Applying this methodology on a three-axle truck creates the complicated computation. Hence, other people also employed discrete model for the truck. For example, Tabatabaei developed a 16-DoF non-linear model an articulated vehicle, which is validated experimentally [30]. It is also possible to reduce air pollutants specially CO2 by acquisition of developed model by optimizing several components in the truck [31, 32].

This paper presents a survey on the equations of motion utilizing Lagrange equations to determine natural frequencies and mode shapes of a three-axle truck, i.e., Benz 2624 model. The main purpose is to investigate the influence of dynamics of the vehicle on its load to determine the maximum speed on different paths. Next, a discussion is provided on the results to distinguish the critical DoF using the mode shapes. The developed 19 DoF model can also be applied on many trucks by changing material properties and adding estimations.

2. Modeling the three-axle truck

Using experimental techniques to obtain mass properties of the components of a manufactured vehicle is the most reasonable but very costly. Hence in this work, Solidworks software is employed to model a three-axle truck and to find masses, centers of mass, moments of inertia etc. These physical properties are highly necessary for dynamic simulation of the truck. Figures 1 and 2 show two views of the assembled model of the truck, and some components of the truck, respectively. In this part, the weighty components of the truck such as chassis, tires, differentials, cabin, springs etc. are modeled precisely. Non homogeneous material is assigned to this model since differential consists of several material and precision of properties, which are obtained from this model and are more acceptable.

![Figure 1. Two views of the CAD model from the three-axle truck.](image)
3. Material Property
Assigning material property to the current model has been very challenging and time consuming. All of the components of truck were modeled separately in order to achieve accurate center of gravity for ADAMS software and initial boundary conditions for MATLAB. Batteries for heavy-duty transport vehicles are so large that the vehicle would barely move, and overhead lines are not practical over millions of acres of farmland, or other off-road logging-trucks, mining trucks, etc., nor could wires be strung over 4 million miles of roads, requiring trucks to have yet another power system after getting off the wires to get to their destination, which doubles the price of the truck. Large, commercial silicon modules convert 17–25% of solar radiation into electricity, and much smaller perovskite cells have already reached a widely reproduced rate of 16–18% in the lab, occasionally spiking higher [33-35]. New Materials for truck can also be assigned to any three axle truck by acquisition of current model.

4. Governing Equations
Lagrange method is utilized to determine dynamic behavior of the mentioned three-axle truck. The truck is considered as a 19-DoF mathematical model. As shown in Figure 3, M₁, M₂ and M₃ are the axles of the truck. Blue springs are considered on behalf of tires, and red springs as leaf springs of the suspensions systems. Also previous mathematical numerical derivation by Zeidi et al. [34-38] has been very useful in this study. Green springs are counted for connecting cabin to the frame and finally, purple spring is used to suspend driver’s seat with respect to the cabin. As the rests, W,θ, and φ illustrate displacement, roll, and pitch of the truck in this dynamic analysis. Hence, the 19 DoFs are as follow:

- Driver seat bounce, one degree; $w_{106}$;
- Cab bounce, pitch and roll, three degrees; orderly $w_{104}$, $θ_{104}$, $ϕ_{104}$;
- Chassis bounce (sprung mass), pitch and roll, three degrees; $w_{100}$, $ϕ_{100}$, $θ_{100}$, respectively;
- Front axle, its bounce and roll, two degrees; orderly $w_{101}$, $θ_{101}$;
- Intermediate axle, bounce and roll, two degrees; orderly $w_{102}$, $θ_{102}$;
- Rear axle, bounce and roll, two degrees; orderly $w_{103}$, $θ_{103}$;
- 6 bounce motion of the 6 wheels; $w₁$, $w₂$, $w₃$, $w₄$, $w₅$, $w₆$; where $w₁$ and $w₂$ are the bounce of left and right steer wheels, respectively; $w₃$ and $w₄$ are the bounce of left and right wheels...
of the middle axle, correspondingly; \( w_5 \) and \( w_6 \) are the bounce of left and right wheels of rear axle, respectively.

The vector of coordinates for the vehicle is written as:

\[
W_{19} = [w_{106} w_{104} \varphi_{104} w_{100} \theta_{100} \varphi_{100} w_{101} \varphi_{101} w_{102} \theta_{102} w_{103} \varphi_{103} w_{1} w_{2} w_{3} w_{4} w_{5} w_{6}]^T
\]  

(1)

Figure 3. The scheme of the 19-DoF model for the truck

Figure 4(a) shows truck model in X-Z plane and distances between different important points. In addition, Figure 4(b) indicates the model in Y-Z plane and the related parameters.
5. Equations of Motion
The Lagrange equation is well-known in the following form for this system:
\[
\frac{d}{dt} \left( \frac{dT}{dW_{19}} \right) - \left( \frac{d^2T}{dW_{19}^2} \right) + \left( \frac{dP}{dW_{19}} \right) = 0
\]  
(2)

Where, T and P are the kinematic and potential energies of the system respectively.

The kinetic energy of the system is as follow:
\[
T = \frac{1}{2} M_c(W'106)^2 + \frac{1}{2} M_c(W'104)^2 + \frac{1}{2} M_b(W'100)^2 + \frac{1}{2} M_1(W'101)^2 + \frac{1}{2} M_2(W'102)^2 + \frac{1}{2} M_3(W'103)^2
+ \frac{1}{2} I_{cx}(\theta'104)^2 + \frac{1}{2} I_{bx}(\theta'100)^2 + \frac{1}{2} I_{1x}(\theta'101)^2 + \frac{1}{2} I_{2x}(\theta'102)^2 + \frac{1}{2} I_{3x}(\theta'103)^2 + \frac{1}{2} I_{cy}(\theta'104)^2
+ \frac{1}{2} I_{by}(\theta'100)^2
\]

Moreover, the potential energy of the system is obtained as:
\[
P = \frac{1}{2} K_c(W106 - W105)^2 + \frac{1}{2} K_c(W45 - W31)^2 + \frac{1}{2} K_c(W46 - W32)^2 + \frac{1}{2} K_c(W47 - W35)^2
+ \frac{1}{2} K_c(W48 - W36)^2 + \frac{1}{2} K_f(W33 - W13)^2 + \frac{1}{2} K_f(W34 - W14)^2 + \frac{1}{2} K_m(W17 - W17)^2
+ \frac{1}{2} K_m(W18 - W18)^2 + \frac{1}{2} K_r(W25 - W25)^2 + \frac{1}{2} K_r(W26 - W26)^2 + \frac{1}{2} K_w(W7 - W1)^2
+ \frac{1}{2} K_w(W8 - W2)^2 + \frac{1}{2} K_w(W9 - W3)^2 + \frac{1}{2} K_w(W10 - W4)^2 + \frac{1}{2} K_w(W11 - W5)^2
+ \frac{1}{2} K_w(W12 - W6)^2
\]

By differentiating of T and P with respect to the coordinates and time according to Eq. (2), equations of motion can be organized as:
\[
M_{19}\ddot{W}_{19} + K_{39}W_{19} = 0
\]
In which $M_{19}$ and $K_{19}$ are orderly mass matrix and stiffness matrix of the 19 DOF of the truck-poster system model. In this equation, $\ddot{w}_{19}$ and $\ddot{w}_{9}$ are acceleration vector and displacement vector of the 19 DOF truck-poster system model. In addition, system mass matrix, $M_{19}$ which is a diagonal matrix, is calculated as follow:

$$M_{19} = \text{diag}[M_1, M_2, M_3, M_4, M_5, M_6, M_9, M_{10}, M_{11}, M_{12}, M_{13}, M_{14}, M_{15}, M_{16}, M_{17}, M_{18}, M_{19}, M_{20}, M_{21}]$$

Where, “diag” illustrates that the $M_{19}$ is a diagonal matrix and $M_8$ to $M_{16}$ are located on the main diagonal of the matrix. In this relation, $M_i$ and $M_j$ are masses of the seat and the driver, and the cab, respectively; $I_{cx}$ and $I_{cy}$ are inertia of the cab about X and Y axes, correspondingly. In the following, $M_0$, $I_{bx}$, and $I_{by}$ point to sprung mass, inertia of the sprung mass about X and Y axes, respectively; Also, 1, 2, 3, and as the indexes in order point to the front axle, middle axle, and the rear axle of the truck. Similarly, $M_{10}$ to $M_{16}$ indicate the masses of the front left to rear right wheels, as well. The 19 nonzero values have been obtained from the truck model in Solidworks software utilizing mass properties. Now, the system stiffness matrix can be written in the following form:

$$K_{19} = \begin{bmatrix} K_{1,1} & K_{1,2} & \cdots & K_{1,19} \\ K_{2,1} & K_{2,2} & \cdots & K_{2,19} \\ \vdots & \vdots & \ddots & \vdots \\ K_{19,1} & K_{19,2} & \cdots & K_{19,19} \end{bmatrix}$$

The non-zero components of $K_{19}$ are as follows:

- $K_{1,1} = K_s$, $K_{1,2} = -K_s$, $K_{1,3} = K_v e_1$, $K_{1,4} = K_s d_1$;
- $K_{2,1} = -K_s$, $K_{2,2} = 4K_c + K_s$, $K_{2,3} = -K_s e_1$, $K_{2,4} = K_c (2d_3 - 2d_2) - K_s d_1$, $K_{2,5} = -4K_c$, $K_{2,7} = K_c (2b_2 + 2b_3)$;
- $K_{3,1} = K_s e_1$, $K_{3,2} = -K_s e_1$, $K_{3,3} = 4K_c e_2^2 + K_s e_1^2$, $K_{3,4} = K_v e_1 d_1$;
- $K_{4,1} = K_s d_1$, $K_{4,2} = -K_s d_1 - 2K_c d_2 + 2K_c d_3$, $K_{4,3} = K_v d_1 e_1$, $K_{4,4} = K_c d_1^2 + 2K_c (d_2^2 + d_3^2)$, $K_{4,5} = 2K_c (d_2 - d_3)$, $K_{4,6} = -2K_c d_3 e_2$, $K_{4,7} = -2K_c d_2 b_4 + 2K_c d_3 b_5$;
- $K_{5,1} = -4K_c$, $K_{5,2} = -2K_c d_1 K_{5,5} = 2K_f + 2K_m + 2K_c + 4K_r$, $K_{5,6} = (2K_f + K_m + K_r) a_1 + 2K_c e_2$, $K_{5,7} = -2K_f b_1 + 2K_m b_2 + 2K_r b_3 - 2K_c b_4$, $K_{5,8} = -2K_r$, $K_{5,10} = -2K_m$, $K_{5,11} = 2K_m a_1 K_{5,12} = -2K_r$;
- $K_{6,2} = -2K_c e_2$, $K_{6,3} = -2K_c e_2^2$, $K_{6,4} = -2K_c e_2 d_3$, $K_{6,5} = 2(K_f + K_m + K_r) a_1 + 2K_c e_2$, $K_{6,6} = 2(K_f + K_m + K_c) a_1^2 + 4K_c e_2^2$, $K_{6,7} = -2K_f a_1 b_1 - 2K_m a_1 b_2 + 2K_r a_1 b_3 - 2K_c e_2 b_4$, $K_{6,8} = -2K_c a_1$, $K_{6,12} = -2K_r a_1$;
- $K_{7,1} = 4K_c b_4$, $K_{7,4} = -2K_c b_4 d_2 + 2K_c b_5 d_3$, $K_{7,5} = -2K_f b_1 + 2K_m b_2 + 2K_r b_3 - 2K_c b_4 - 2K_c b_5$, $K_{7,6} = -2K_f a_1 b_1 + 2K_m a_1 b_3 + 2K_c e_2 b_4$, $K_{7,7} = 2K_f b_1^2 + 2K_m b_2^2 + 2K_r b_3^2 + 2K_c b_4^2 + 3K_c b_5^2$, $K_{7,8} = 2K_r b_1$, $K_{7,10} = -2K_m b_2$, $K_{7,11} = 2K_m b_3$, $K_{7,12} = -2K_r b_3$;
- $K_{8,1} = -2K_c a_1$, $K_{8,2} = 2K_f b_1$, $K_{8,8} = 2K_f + 2K_w r$;
- $K_{9,1} = 2K_w r a_2^2 + 2K_1 a_1^2 K_{9,14} = -K_w r a_2$, $K_{9,15} = K_w r a_2$;
- $K_{10,5} = -2K_m$, $K_{10,6} = -2K_m a_1$, $K_{10,7} = -2K_m b_3$, $K_{10,10} = 2K_w r$, $K_{10,11} = -2K_m a_1$, $K_{10,16} = -K_w r$, $K_{10,17} = -K_w r$;
- $K_{11,5} = 2K_m a_1$, $K_{11,7} = 2K_m b_2 a_1$, $K_{11,10} = -2K_m a_1$, $K_{11,11} = 2K_m a_1^2 + 2K_w r a_2^2$, $K_{11,16} = -K_w r a_2$, $K_{11,17} = K_w r a_2$;
- $K_{12,5} = -2K_f$, $K_{12,6} = -2K_5 a_1$, $K_{12,7} = -2K_f b_3$, $K_{12,12} = 2K_w r$, $K_{12,18} = -2K_w r$;
- $K_{13,13} = 2K_w r a_2^2 + 2K_5 a_1^2$, $K_{13,18} = -K_w r a_2$, $K_{13,19} = -K_w r a_2$;
- $K_{14,8} = -K_w r$, $K_{14,9} = -K_w r a_2$, $K_{14,14} = K_w r$;
- $K_{15,8} = -K_w r$, $K_{15,9} = K_w r a_2$, $K_{15,15} = K_w r$.
In which, $k_s$ is the stiffness of spring of driver seat; $K_c$ is the stiffness of each spring of cab suspension; $K_f$ is the stiffness of each spring of front axle suspension; $K_m$ and $K_r$ are defined for stiffness of every spring for the middle axle and the rear axle, respectively; $K_wf$ is also considered for the equivalent stiffness of each of front tires, while $K_wf$ plays the same role for the tires of middle and rear wheels. Other variables and constants were illustrated in the previous sections.

6. Validation

ADAMS software has provided a unique environment for dynamic modeling of multibody systems. Here, we utilize the software to validate the equations of motion for the truck. For this purpose, one of the modules of this software, i.e., ADAMS/Truck, is employed. The three-axle truck model, which is available in the software, is utilized and the characteristics of the truck are entered. The truck, as a vibrational system is moved on a sinusoidal road to excite the model with a wide range of frequencies as shown in Figure 5. Times, in which the accelerations of the CG are more than their neighborhood, can be used to find natural frequencies using constant acceleration of the truck. Figure 6 shows accelerations obtained from this procedure.

![Sinusoidal Road](image.png)

**Figure 5. Calculating truck dynamic characteristics using ADAMS**
Figure 6. Acceleration for the truck in ADAMS

To verify the accuracy of the results, some of the natural frequencies, which are derived from MATLAB code, are compared with the simulation results from ADAMS software as shown in table 1. Comparing two or three natural frequencies of the system with those obtained from analytical procedure replies that the formulation is valid.

<table>
<thead>
<tr>
<th>Dominant motion</th>
<th>Natural frequency(rad/s) (Analytically)</th>
<th>Natural frequency(rad/s) (ADAMS)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right steer wheel bounce</td>
<td>77.1316</td>
<td>83.1044</td>
<td>7.18</td>
</tr>
<tr>
<td>Rear axle bounce</td>
<td>103.358</td>
<td>90.1399</td>
<td>12.78</td>
</tr>
<tr>
<td>Center axle bounce</td>
<td>265.249</td>
<td>261.174</td>
<td>1.56</td>
</tr>
</tbody>
</table>

7. Natural frequencies and corresponding mode shapes
To determine the natural frequencies of the system, the free vibration of the system is considered. The equations of motion for free vibration were derived in eq. 5. The characteristic equation of the system is arisen from eq. 5 and Eigen values of the characteristic equation are natural frequencies of the system. Eigenvectors of the equation are utilized to find natural frequency corresponding to each DOF.
Table 2 shows, natural frequencies and mode shapes for the 19 DOF truck. For each natural frequency, there is a 19×1 vector of mode shape. Among the elements of every mode shape, the highest value shows dominant motion. Dominant motion is the most intense motion in each natural frequency.

<table>
<thead>
<tr>
<th>Natural frequency(rad/s)</th>
<th>Dominant motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000004622358</td>
<td>right rear steer wheel bounce</td>
</tr>
<tr>
<td>0.000003451677</td>
<td>right steer wheel bounce</td>
</tr>
<tr>
<td>0.000000001065513</td>
<td>left rear steer wheel bounce</td>
</tr>
<tr>
<td>0.011396794116965</td>
<td>Deriver seat bounce</td>
</tr>
<tr>
<td>0.022510398030284</td>
<td>Cab roll</td>
</tr>
<tr>
<td>0.0326681119888841</td>
<td>Cab bounce</td>
</tr>
<tr>
<td>50.670520926536</td>
<td>right rear steer wheel bounce</td>
</tr>
<tr>
<td>51.637956185048</td>
<td>left center steer wheel bounce</td>
</tr>
<tr>
<td>64.379923963352</td>
<td>left center steer wheel bounce</td>
</tr>
<tr>
<td>66.62659703922</td>
<td>Cab roll</td>
</tr>
<tr>
<td>70.244931281563</td>
<td>Left steer wheel bounce</td>
</tr>
<tr>
<td>77.136046229719</td>
<td>Right steer wheel bounce</td>
</tr>
<tr>
<td>103.358018471211</td>
<td>Rear axle bounce</td>
</tr>
<tr>
<td>189.294739253455</td>
<td>Center axle roll</td>
</tr>
<tr>
<td>189.588958526633</td>
<td>Center axle roll</td>
</tr>
<tr>
<td>265.249162539134</td>
<td>Center axle bounce</td>
</tr>
<tr>
<td>586.273194921182</td>
<td>Steer axle bounce</td>
</tr>
<tr>
<td>596.770119450067</td>
<td>Steer axle roll</td>
</tr>
<tr>
<td>1133.920811119840</td>
<td>Steer axle bounce</td>
</tr>
</tbody>
</table>

8. Natural frequency using 16 DoF model
In general, the DOF of the driver seat and the cab of the truck are not considered. To investigate the influence of this withdrawal, by excluding three degrees of freedom – driver seat bounce, cab bounce and cab roll- a 16 DOF model is provided and solved. Hence, three dominant motions from three different ranges are opted to compare from those that were obtained from 19 DOF model. According to table 3, by eliminating some degrees of freedom, natural frequencies aggravate. Comparing the results reports about 10% error in natural frequencies.

Table 3. Natural frequencies using 16 DoF model for three dominant motions

<table>
<thead>
<tr>
<th>Natural frequency(rad/s)</th>
<th>Dominant motion</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>53.874560935562</td>
<td>right rear steer wheel bounce</td>
<td>6.3%</td>
</tr>
<tr>
<td>201.494719652476</td>
<td>Center axle roll</td>
<td>6.5%</td>
</tr>
<tr>
<td>1250.354821459930</td>
<td>Steer axle bounce</td>
<td>10.4%</td>
</tr>
</tbody>
</table>

9. Calculation of the Suspension Static Deflection and the Reaction Force on Each Wheel
The suspension static displacement which is the relative displacement of the two ends of the suspension can be obtained as following (for instance we have just mentioned seat and cab static displacement formula in which the subscripts are f = front, r=right, l = left, re = rear):

\[ d_{\text{seat}} = w_{106} - w_{105} = w_{106} - (w_{104} - d_1\theta_{104} - e_1\phi_{104}) \]

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The displacements of all degrees of freedom that have been used above have been calculated by considering the gravity. Hence, the static deflection of each suspension spring member is listed below:

<table>
<thead>
<tr>
<th>Static Deflection (m)</th>
<th>Seat</th>
<th>Cab, front left</th>
<th>Cab, front right</th>
<th>Cab, rear left</th>
<th>Cab, rear right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.0594</td>
<td>-0.0088</td>
<td>-0.0212</td>
<td>-0.0092</td>
<td>-0.0216</td>
</tr>
<tr>
<td></td>
<td>Main, steer left</td>
<td>Main, steer right</td>
<td>Main, center left</td>
<td>Main, center right</td>
<td>Main, rear left</td>
</tr>
<tr>
<td></td>
<td>-0.1875</td>
<td>-0.1895</td>
<td>-0.1456</td>
<td>-0.1488</td>
<td>-0.1173</td>
</tr>
<tr>
<td></td>
<td>Tire, steer left</td>
<td>Tire, steer right</td>
<td>Tire, center left</td>
<td>Tire, center right</td>
<td>Tire, rear left</td>
</tr>
<tr>
<td></td>
<td>-0.031</td>
<td>-0.0311</td>
<td>-0.0348</td>
<td>-0.0351</td>
<td>-0.0279</td>
</tr>
</tbody>
</table>

And the reaction force on each wheel can be obtained by considering tire stiffness; hence result for the reaction force calculation is listed in the following table:

<table>
<thead>
<tr>
<th>Reaction force (N)</th>
<th>Tire, steer left</th>
<th>Tire, steer right</th>
<th>Tire, center left</th>
<th>Tire, center right</th>
<th>Tire, rear left</th>
<th>Tire, rear right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21399</td>
<td>21483</td>
<td>48102</td>
<td>48531</td>
<td>38525</td>
<td>38954</td>
</tr>
</tbody>
</table>

It is clearly seen that wheels on the right side are subject to slightly greater loads than those on the left side consistent with the chassis and cab lean. This effect results from the offset position of the driver seat.

10. Conclusion
A 19-DoF system had been chosen in order to study the dynamic characteristics of a three-axle truck. Physical properties are calculated using a model in Solidworks CAD software. Lagrange equations are used for deriving equations of motion. Validation performed by comparison of data achieved analytically and those resulted from ADAMS simulation, which replies the accuracy of the formulation. Finally, natural frequencies, mode shapes and dominant motions have been observed. In each natural frequency, a 19-dimensional mode shape is established, which is useful to know the vibrating component of the truck. Finally, the influence of elimination of three non-prominent DOFs is investigated. The results illustrate that the equations of motion obtained in this work can
increase the accuracy by 10%. It was also found that right side wheels of the truck experience more intense load in comparison with the left side wheels due to chassis and cab lean.

11. References


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