A New Temper Rolling Force Model for Dry Thick Strip Considering Fully Elastic Contact

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Abstract
Temper rolling or skin pass rolling is one of cold rolling processes composed of a horizontal pass cold rolling mill stand. In this process, the thick strip is subjected to very light reduction (0.5–4%) in thickness in the presence of high friction amplitudes. One of the basic parameters of strip rolling processes is temper rolling force. A new rolling force model has been proposed for temper rolling strip mill considering the inhomogeneous stress distribution along the thickness of the strip. The contact between rolls and thick strip is totally elastic. The deformation field consists of an elastic (actually elastic) zone and that is highly non-homogeneous across the thickness of the strip. Totally elastic formulation can be used to predict the roll force. It can be used to study the stress distribution along the thickness of strip. The computing results from this model coincide the similar presented results in previous investigations done.

Keywords
Temper rolling mill, rolling force, elastic contact, strip

1. Introduction

A temper mill is a steel sheet or steel plate processing line composed of a horizontal pass cold rolling mill stand, entry and exit conveyor tables or mandrills and upstream and downstream equipment depending on the design and nature of the processing system. But rolling force calculation models have not been adequately investigated [1-4]. In many rolling calculations, scientists naturally extended the rolling force models of cold rolling to the calculation of temper rolling force for the temper rolling process with a thick strip. In Baoshan Iron & Steel Ltd, using a hot temper rolling mill, the tests of temper rolling force are carried out. It is indicated that the traditional models are not so accurate to calculate ones [5-8]. This could be due to the Karman plane that should consider adopted models and the stress which was taken homogeneous in the deformation zone distribution, with low elongation and small plastic deformation there is an inhomogeneous stress distribution along the thickness of the strip. So, Ref. [5] presented a new model for the stress distribution along the thickness in the deformation zone and in the condition of totally elastic contact between roll and thick strip. Considering totally elastic contact, new model was applied to the calculation of stress in elastic deformation with no plastic deformation in the present study. Finally, an analytical model based on an exponential rolling pressure function considering inhomogeneous stress distribution along the thickness of strip has been proposed.

2. Inhomogeneous stress distribution along the thickness in a state of elastic contact between strip and roll

Using Fourier progression in a limited length of strip according to the mechanics of elasticity plane problems, a distributing load was assumed to be applied on the upper and lower surfaces of the strip and roll. Stress parameters derived from the stress functions [8]. In the cold rolling processes, a rolling force p(x) distributing in an exponential function form within the contact length has been proposed.

Note that this equation is an exponential model in determining the rolling force in the rolling processes (Figure 1).

$$p(x) = p_0 e^{\frac{2\mu}{h}x} \tag{1}$$

Where h is the average thickness during the rolling and $h = h_m = \frac{h_f+h_0}{2}$.$h_f$ and h_0 are the output and input strip thicknesses respectively.

So we have:

$$p(x) = p_0 e^{\frac{2\mu}{h_m}x} \tag{2}$$

Firstly, the normal stress $p(x)$ on the surface of the strip was expended to Fourier progression within a limited length of $(-L/2, L/2)$

$$p(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L} \tag{3}$$

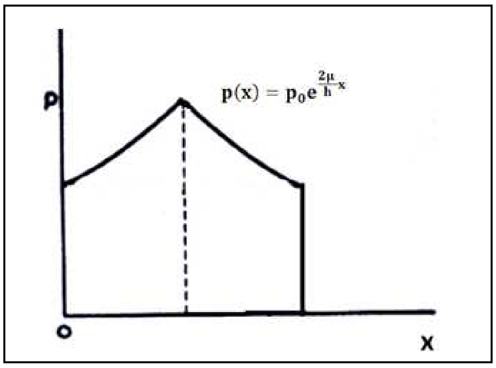

Figure1. Rolling force distribution

For the load is symmetry to Y axis, so b_n= 0, then, the load is expressed as:

$$p(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} \tag{4}$$

Where

$$a_0 = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} p(x)dx = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} p_0 e^{\frac{2\mu}{h_m}x} dx = \frac{p_0}{\frac{2\mu}{h_m}L} e^{\frac{2\mu}{h_m}L} \tag{5}$$

,

$$a_n = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} p(x)\cos\frac{n\pi x}{L}dx = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} p(x)\cos\frac{n\pi x}{L}dx = \frac{p_0}{L}\left[\frac{\left(-\frac{4\mu}{h_m}\cos\frac{n\pi}{L}+\frac{n\pi}{L}\sin\frac{n\pi}{L}\right)\times\left(\frac{e^{\frac{2\mu}{h_m}}-1}{e^{\frac{\mu}{h_m}}}\right)}{\left(\left(\frac{2\mu}{h_m}\right)^2+\left(\frac{n\pi}{L}\right)^2\right)}\right] \quad (6)$$

The first item located in the right of Eq. (6) was the average of $p(x)$ in the whole length of strip. It was corresponding to the situation which the strip was imposed by homogeneous compressing along y direction (σ_y=p(x), $\sigma_y = \tau_{xy} = 0$). Next is to get the stress components corresponding to second item of $p(x)$.Because there is a symmetrical load distribution with aspect to the half of the strip thickness, the stress function was chosen as:

$$\Psi(x,y) = f_n(y)\cos\frac{n\pi x}{L} \quad , \; n = 1,2,3,\ldots \quad (7)$$

Let $k_n = \frac{n\pi}{L}$ $\quad So \quad$ $\Psi(x,y) = f_n(y)\cos k_n x$ (8)

With Inserting and solving it in harmonized equation, stress function was given as:

$$\nabla^4\Psi = 0 \quad (9)$$

$$\Psi(x,y) = \sum_{n=1}^{\infty}\left(A_n cosh(k_n y) + B_n sinh(k_n y) + C_n y cosh(k_n y) + D_n y sinh(k_n y)\right) \quad (10)$$

Consequently, the corresponding stress components were shown below:

$$\sigma_X = \frac{\partial^2\Psi}{\partial y^2} = \sum_{n=1}^{\infty} {k_n}^2\left(A_n cosh(k_n y) + B_n sinh(k_n y) + \frac{2}{k_n}C_n y cosh(k_n y) + \frac{2}{k_n}D_n y sinhk_n y\right)\cos k_n x \quad (11)$$

$$\sigma_y = \frac{\partial^2\Psi}{\partial x^2} = -\sum_{n=1}^{\infty} {k_n}^2 \left(A_n cosh(k_n y) + B_n sinh(k_n y) + C_n y cosh(k_n y) + D_n y sinhk_n y\right)\cos k_n x \quad (12)$$

$$\tau_{xy} = -\frac{\partial^2\Psi}{\partial x\partial y} = \sum_{n=1}^{\infty} {k_n}^2\left(A_n sinh(k_n y) + B_n cosh(k_n y) + \frac{C_n}{k_n}cosh(k_n y) + C_n y sinh(k_n y) + \frac{D_n}{k_n}sinh(k_n y) + D_n y cosh(k_n y)\right) \quad (13)$$

Where indefinite parameters A_n, B_n, C_n, D_n can be defined by the following conditions.

When

$$y = \pm\frac{h}{2}, \sigma_y = \sum_{n=1}^{\propto} {k_n}^2 \left(A_n coshk_n y + D_n ysinhk_n y\right)\cos k_n x =$$

$$\frac{p_0}{\frac{2\mu}{h_m}L} e^{\frac{2\mu}{h_m}L} + \frac{p_0}{L}\left[\frac{\left(-\frac{4\mu}{h_m}\cos\frac{n\pi}{L} + \frac{n\pi}{L}\sin\frac{n\pi}{L}\right)\times\left(\frac{e^{\frac{2\mu}{h_m}}-1}{\frac{\mu}{e^{h_m}}}\right)}{\left(\left(\frac{2\mu}{h_m}\right)^2 + \left(\frac{n\pi}{L}\right)^2\right)}\right], \tau_{xy} = 0 \qquad (14)$$

For distributing loads on the upper and lower surface of strip to be totally equal, σ_y must be symmetrical to *x*-axis. Then the coefficient of dissymmetry function $sinh(k_n y)$ and $ycoshk_n y$ in the expression of σ_y in Eq.(10) must be zero. So, B_n and C_n are zero.

Inserting them in the expression of σ_y and τ_{xy} and mixing with boundary conditions give:

$$D_n = \frac{\frac{p_0 h}{2\mu} e^{\frac{2\mu}{h_m}} + \frac{p_0}{L}\left(\frac{\frac{n\pi}{L}}{\left(\left(\frac{2\mu}{h_m}\right)^2 + \left(\frac{n\pi}{L}\right)^2\right)}\right)\left(\frac{e^{\frac{2\mu}{h_m}}-1}{\frac{\mu}{e^{h_m}}}\right)}{{k_n}^2\left(\frac{1}{k_n} + \frac{h}{2}\left(\frac{1+2sinh\left(k_n\frac{h}{2}\right)^2}{sinh\left(k_n\frac{h}{2}\right)}\right)\right)} \qquad (15)$$

$$A_n = \left[\frac{\frac{p_0 h}{2\mu} e^{\frac{2\mu}{h_m}} + \frac{p_0}{L}\left(\frac{\frac{n\pi}{L}}{\left(\left(\frac{2\mu}{h_m}\right)^2 + \left(\frac{n\pi}{L}\right)^2\right)}\right)\left(\frac{e^{\frac{2\mu}{h_m}}-1}{\frac{\mu}{e^{h_m}}}\right)}{{k_n}^2\left(\frac{1}{k_n} + \frac{h}{2}\left(\frac{1+2sinh\left(k_n\frac{h}{2}\right)^2}{sinh\left(k_n\frac{h}{2}\right)}\right)\right)}\right] \times \left(\frac{1}{k_n} + \frac{h}{2}coth\left(k_n\frac{h}{2}\right)\right) \qquad (16)$$

After above coefficients were inserted in Eq.(11) and replaced k_n with $\frac{n\pi}{L}$, the distribution of σ_X along the thickness of strip was given below:

$$\sigma_X(y) = \frac{\frac{p_0 h}{2\mu} e^{\frac{2\mu}{h_m}} + \frac{p_0}{L}\left(\frac{\frac{n\pi}{L}}{\left(\left(\frac{2\mu}{h_m}\right)^2 + \left(\frac{n\pi}{L}\right)^2\right)}\right)\left(\frac{e^{\frac{2\mu}{h_m}}-1}{\frac{\mu}{e^{h_m}}}\right)}{{k_n}^2\left(\frac{1}{k_n} + \frac{h}{2}\left(\frac{1+2sinh\left(k_n\frac{h}{2}\right)^2}{sinh\left(k_n\frac{h}{2}\right)}\right)\right)} \times \sum_{n=1}^{\propto} {k_n}^2 \left\{\left(\frac{1}{k_n} + \frac{h}{2}coth\left(k_n\frac{h}{2}\right)\right)cosh(k_n y) + \right.$$

$$\left. ysinh(k_n y)\right\}\cos\frac{n\pi x}{L} \qquad (17)$$

Moreover, the value of σ_{xn0} on the entry section in deformation zone was gained when $l = \frac{1}{2}(\frac{Dr\mu}{2} + \sqrt{\left(\frac{Dr\mu}{2}\right)^2 + 2Dtr})$. l is the length of the rolling bite. $x = \pm\frac{l}{2}$and $y = \frac{h_0}{2}$ where R is the work roll radius and the Δh reduction is $\Delta h = h_0 - h_f$[6]. Because of the low deformation in the temper mill rolling in comparison with the rolling stand mills, we assume that the work rolls remains rigid.

The distribution of σ_{xn0} on the entry section of deformationzone along the thickness was solved by an example of p = 443.15 N, h = 4.366 mm, l = 5.38 mm and L = 1000 from Ref. [6] (Figure 2).

$$\sigma_{xn0} = \sigma_X(y) = \frac{\frac{p_0 h}{2\mu} e^{\frac{2\mu}{h_m}} + \frac{p_0}{L}\left(\frac{\frac{n\pi}{L}}{\left(\left(\frac{2\mu}{h_m}\right)^2 + \left(\frac{n\pi}{L}\right)^2\right)}\right)\left(\frac{e^{\frac{2\mu}{h_m}-1}}{e^{\frac{\mu}{h_m}}}\right)}{k_n{}^2\left(\frac{1}{k_n} + \frac{h}{2}\left(\frac{1+2sinh\left(k_n\frac{h}{2}\right)^2}{sinh\left(k_n\frac{h}{2}\right)}\right)\right)} \times$$

$$\sum_{n=1}^{\propto} k_n{}^2\left\{\left(\frac{1}{k_n} + \frac{h}{2} coth\left(k_n\frac{h}{2}\right)\right) cosh\left(k_n\frac{h_0}{2}\right) + \right.$$

$$\left.\frac{h_0}{2} sinh\left(k_n\frac{h_0}{2}\right)\right\} \cos\frac{-n\pi\frac{1}{2}\left(\frac{Dr\mu}{2} + \sqrt{\left(\frac{Dr\mu}{2}\right)^2 + 2Dtr}\right)}{L} \tag{18}$$

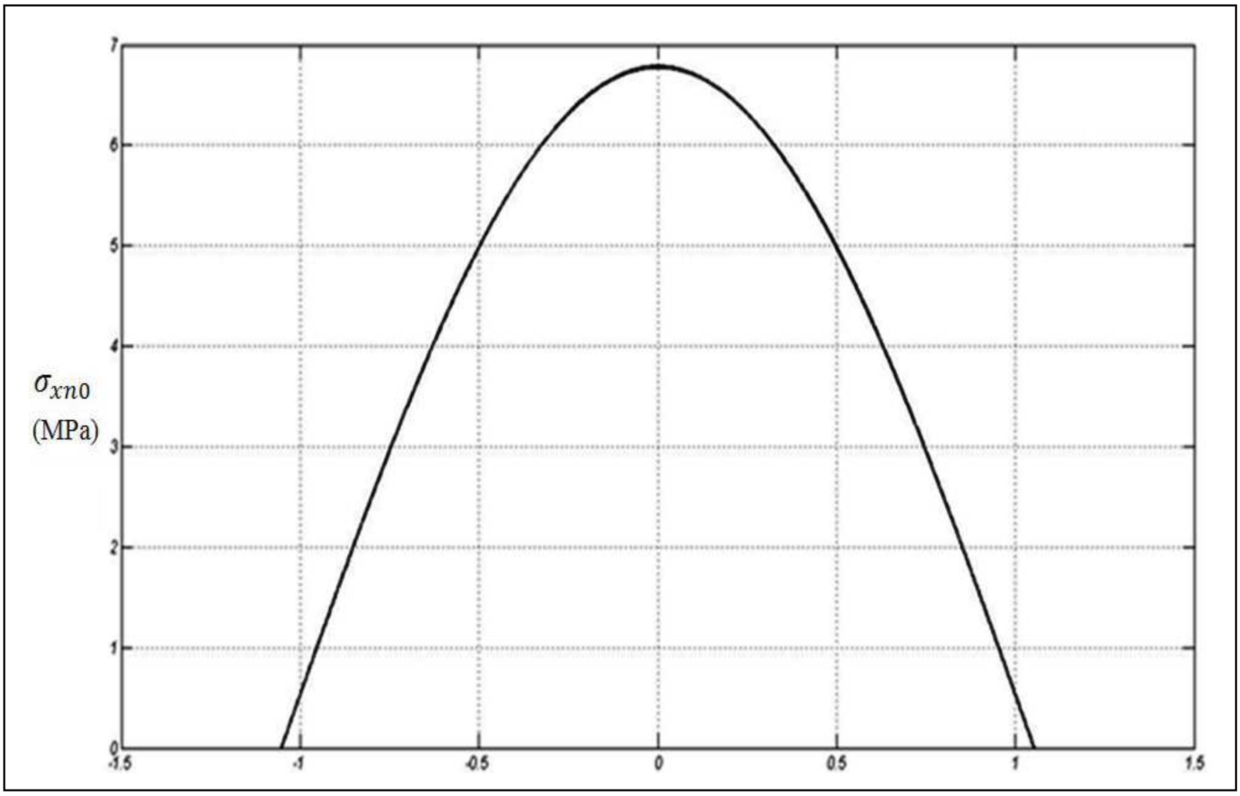

Figure 2. Distribution of σ_{xn0} along the thickness on the strip entry section

3. The temper rolling force model

The deformation zone during temper rolling should be similar with that of critical elastic contact which is inhomogeneous along the thickness of strip for low elongation in temper rolling. Therefore, it is necessary to adopt the boundary conditions of contact surface between roll and strip on the condition of not adopting the Karman's plane assumption.

$$p_x \cos\phi_x \pm t_x \sin\phi_x = \sigma_{yn}\cos\phi_x \mp \tau_{xn}\sin\phi_x \quad (19)$$
$$p_x sin\phi_x \mp t_x cos\phi_x = \sigma_{xn} sin\phi_x \mp \tau_{xn} cos\phi_x \quad (20)$$

Combining these boundary conditions with the following equilibrium differential equations by writing the equilibrium equations, we see that the stresses equations satisfy the equilibrium equations. [5]

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} = 0 \quad (21)$$
$$\frac{\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} = 0 \quad (22)$$

And the hypothesis of linear distribution of τ_{xy}along thickness is

$$\tau_{xy} = 2\eta(v_n - v_0)\frac{y}{{y_n}^2} \quad (23)$$

The relational expression of $\sigma_{xy}$and τ_{xy}on the contact surface was given as:

$$\frac{d\sigma_{xn}}{dx} = \pm\frac{\tau_{xn}}{y_n} \quad (24)$$

When temper rolling was regarded as the compression between two parallel planes,
$\phi_x = 0$,$cos\phi_x = 1$, $sin\phi_x = 0$ Then $\tau_{xn} = t_x$.

If it is Inserted in Eq. (24) and let $y_n = h/2$, then:

$$\frac{d\sigma_{xn}}{dx} = \pm\frac{2t_x}{h} \quad (25)$$

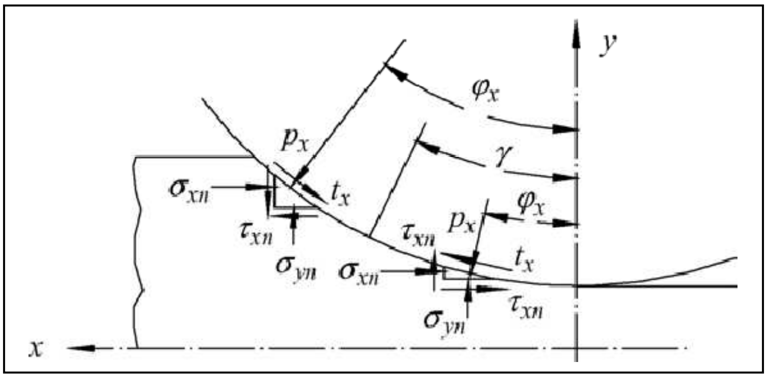

Figure3. Boundary conditions of forward and backward slip zone on the contact deformation zone [5]

This equation was inserted in literature [5], changed $\sigma_0$and $\sigma_1$and previous horizontal stress on the boundary of entry and exited in deformation zone into $\sigma 0$ +σ_{xn0} and σ_1+σ_{xn0}.Here, σ_{xn0} was the stress value of contact surface.

In entry section, it is resulted from rolling force and friction on the contact surface together. σ_{xn0} can be calculated by the formula indicated in the last section considering the stress distribution on entry. Exit section on the condition of elastic contact can be adopted as the stress distributing on entry and it exits section during temper rolling process for low plastic deformation. Moreover, the temper rolling force could also be calculated through looking up the deformation resistance corresponding to the deformation rate on the contact surface, ε_n[9, 10].

4. Validation of the proposed model

The validity of this model for calculating temper-rolling force has been proved by comparison with Ref. [5]. The parameters of material, rolling process and the contrast between calculated and experimental rolling force are listed in Tables 1, 2.

Table1. Parameters of material and rolling process

Number of strip	Material of strip	Specification (mm)	T.S./Y.S. (MPa)	Back tension (KN)	Front tension (KN)	Rolling speed (m/min)
1	SPHC	4. 50×1495	335/220	54	130	190
3	SS400	3. 0×1490	470/320	54	130	190
	B480GNQR	2. 35×1300	545/390	54	130	200

Table2. Difference between calculated and experimental rolling force

Number of strip	Measured elongation (%)	Deformation on the contact surface (MPa)	Friction coefficient in calculation (dry)	Measured rolling force setting value of oil pressure (T)	Calculated value with literature [9]	Calculated value with literature[5]	Present study
1	0.96	310	0.2	300	269.5	310.5	300.5
2	0.643	380	0.2	300	264.2	305.6	294.1
3	0.8	410	0.2	300	251.2	293.2	287.2

Table 2 shows that the total rolling force similar to actual case can be calculated by this rolling force model for temper rolling. The inhomogeneous stress distribution along the thickness of strip in the deformation zone was taken into account in this rolling force model for temper rolling mill. This model proved to be valid by references [5, 9] and can be used to calculate the temper rolling force after taking into account friction coefficient and deformation resistance.

5. Conclusion

For improving calculation accuracy of an elastic contact in cold rolling in tandem mill, a new analytical model for the inhomogeneous stress distribution along the thickness of strip in the deformation zone was taken into account in this study. The new model proved to be valid by experiment results and it can be used to calculate the temper rolling force.

6. References

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